Geometry

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Bjarki Ágúst Guðmundsson
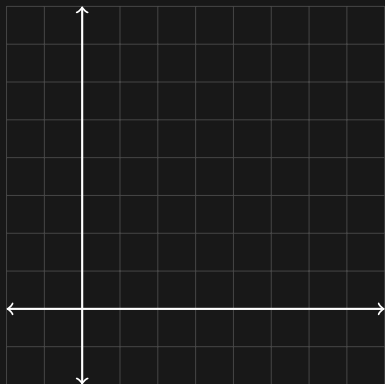
School of Computer Science
Reykjavík University

Árangursrík forritun og lausn verkefna
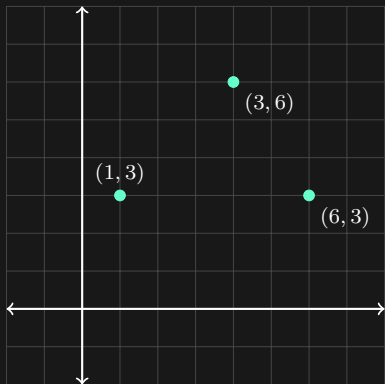
Today we're going to cover

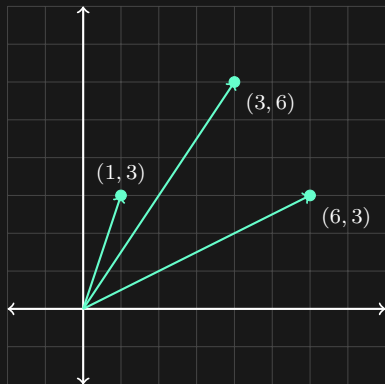- Geometry
- Computational geometry

Points and vectors

Points and vectors
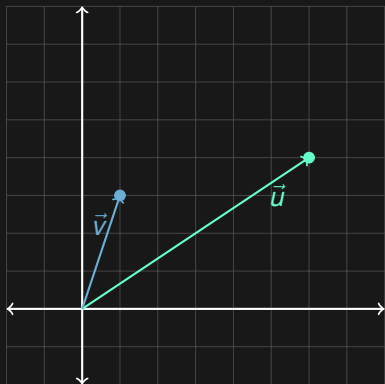

- Points are represented by a pair of numbers, (x, y).
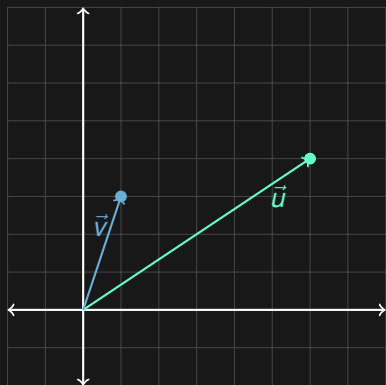
Points and vectors

- ▸ Points are represented by a pair of numbers, (x, y).
- ▸ Vectors are represented in the same way.
- ▸ Thinking of points as vectors us to do many things.

Points and vectors

Points and vectors

- Simplest operation, addition is defined as

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 + x_1 \\ y_0 + y_1 \end{pmatrix}$$

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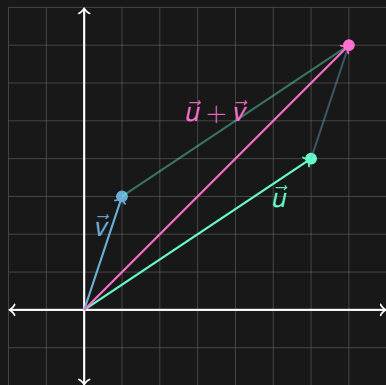
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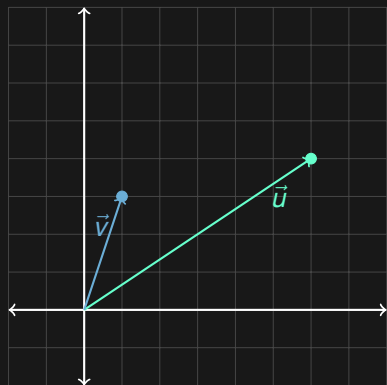
- Subtraction is defined in the same manner

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 - x_1 \\ y_0 - y_1 \end{pmatrix}$$

Points and vectors
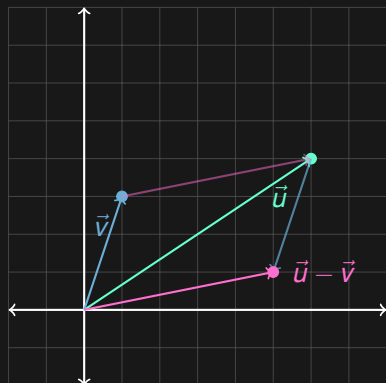

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Points and vectors

```
struct point {
    double x, y;
    point(double _x, double _y) {
        x = _x, y = _y;
    }

    point operator+(const point &oth){
        return point(x + oth.x, y + oth.y);
    }

    point operator-(const point &oth){
        return point(x - oth.x, y - oth.y);
    }
};
```

Points and vectors

…or we could use the `complex<double>` class.

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typedef complex<double> point;
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The `complex` class in `C++` and `Java` has methods defined for

- ▸ Addition
- ▸ Subtraction
- ▸ Multiplication by a scalar
- ▸ Length
- ▸ Trigonometric functions

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Complex numbers have the real part and the imaginary part. Can be thought of as vectors or points on the complex plane.

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- ▸ `double real(p)` returns the real part, or in our case, the *x* value of `p`

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- ▶ `double imag(p)` returns the imaginary part, *y* value of `p`.
- ▶ `double abs(p)` returns the absolute value of the complex number, the length of the vector.
- ▶ `double sin(p)`, `double cos(p)`, `double tan(p)`, trigonometric functions.
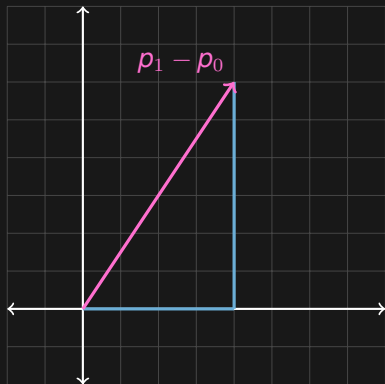
Lines and line segments
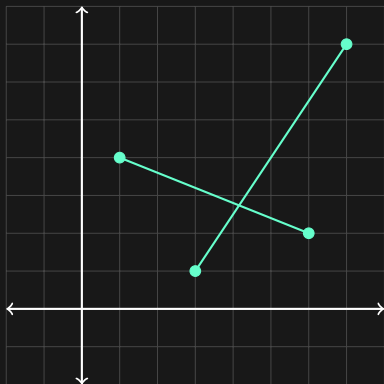
▶ Line segments are represented by a pair of points, $((x_0, y_0), (x_1, y_1))$.

Lines and line segments
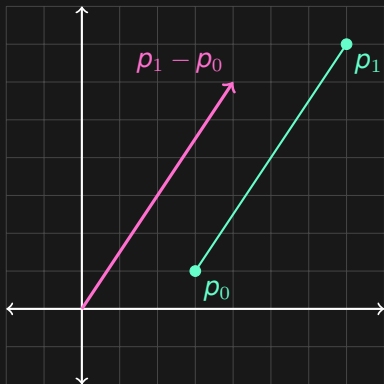
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- Distance between two points is the length of the line segment or vector between the points.

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$$d((x_0, y_0), (x_1, y_1))$$
$$= |(x_1 - x_0, y_1 - y_0)|$$
$$= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

Lines and line segments
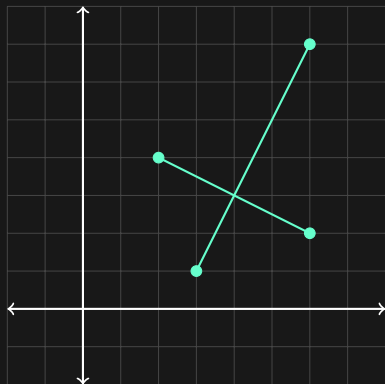
```cpp
struct point {
    ...
    double distance(point oth = point(0,0)) const {
        return sqrt(pow(x - oth.x, 2.0)
                  + pow(y - oth.y, 2.0));
    }
    ...
}
```
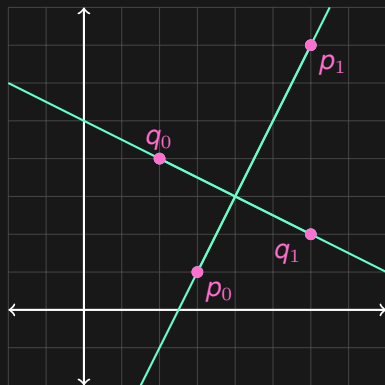
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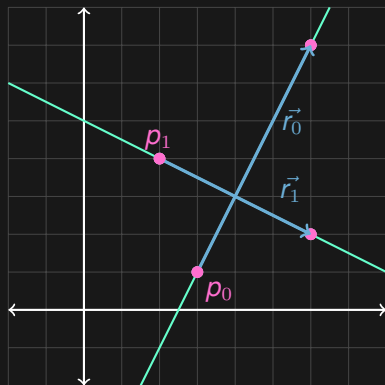Or use the `abs` function with `complex<double>`.

Lines and line segments

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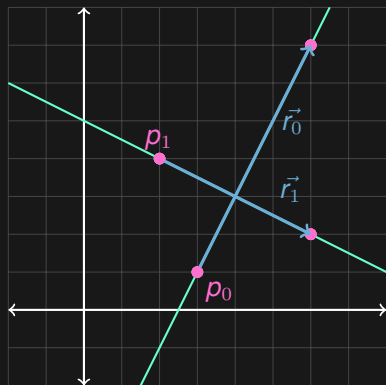
Lines and line segments

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- Or as a point and a direction vector.
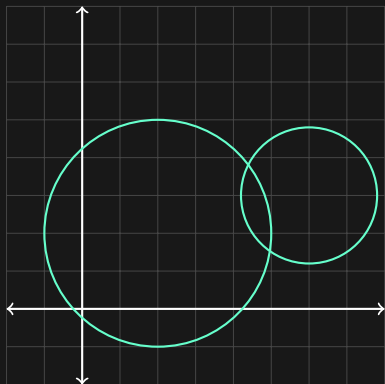
$$p + t \cdot \vec{r}$$

Lines and line segments

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$$p + t \cdot \vec{r}$$

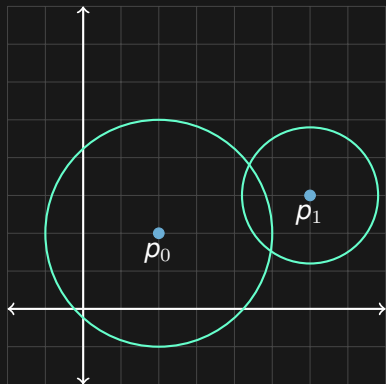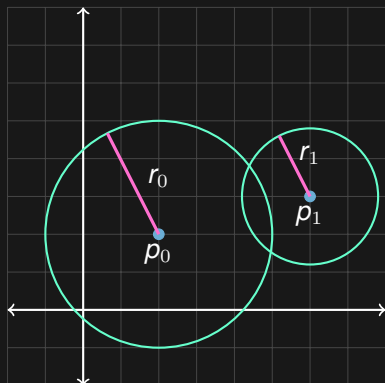- Either way
  `pair<point,point>`

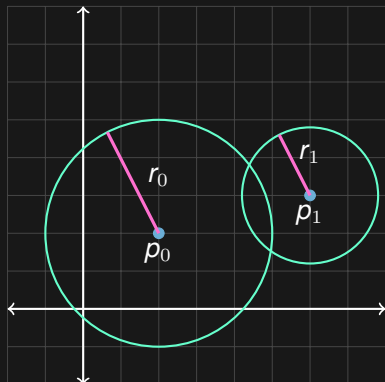# Circles

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  `pair<point,double>`

Dot product

Given two vectors

$$\vec{u} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

the dot product of $\vec{u}$ and $\vec{v}$ is defined as

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = x_0 \cdot x_1 + y_0 \cdot y_1$$

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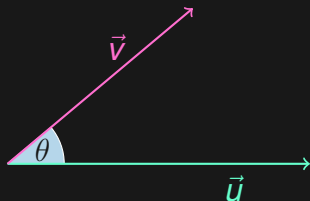the dot product of $\vec{u}$ and $\vec{v}$ is defined as

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Which in geometric terms is

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta$$

Dot product

▸ Allows us to calculate the angle between $\vec{u}$ and $\vec{v}$.

$$\theta = \arccos\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)$$

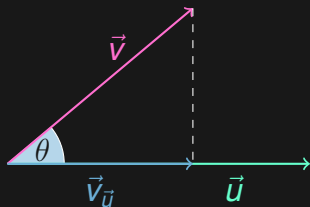# Dot product

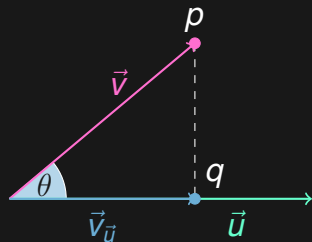- Allows us to calculate the angle between $\vec{u}$ and $\vec{v}$.

$$\theta = \arccos\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)$$

- And the projection of $\vec{v}$ onto $\vec{u}$.

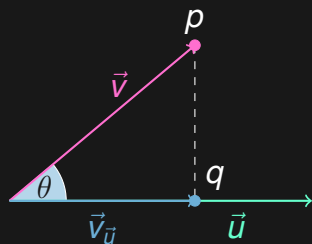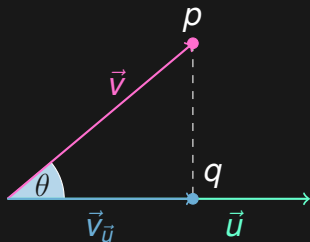$$\vec{v}_{\vec{u}} = \left(\frac{\vec{u} \cdot \vec{v}}{|u|}\right)\vec{u}$$

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- The distance from p to $\vec{u}$ is the distance from p to q.
- Unless q is outside $\vec{u}$, then the closest point is either of the endpoints.

Dot product

Rest of the code will use the `complex` class.

```
#define P(p) const point &p
#define L(p0, p1) P(p0), P(p1)
double dot(P(a), P(b)) {
    return real(a) * real(b) + imag(a) * imag(b);
}
double angle(P(a), P(b), P(c)) {
    return acos(dot(b - a, c - b) / abs(b - a) / abs(c - b));
}
point closest_point(L(a, b), P(c), bool segment = false) {
    if (segment) {
        if (dot(b - a, c - b) > 0) return b;
        if (dot(a - b, c - a) > 0) return a;
    }
    double t = dot(c - a, b - a) / norm(b - a);
    return a + t * (b - a);
}
```

Cross product

Given two vectors

$$\vec{u} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

the cross product of $\vec{u}$ and $\vec{v}$ is defined as

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Cross product

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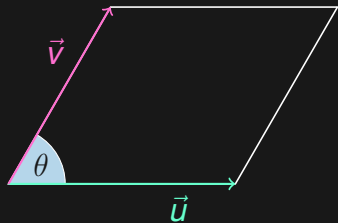the cross product of $\vec{u}$ and $\vec{v}$ is defined as

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Which in geometric terms is

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin \theta$$
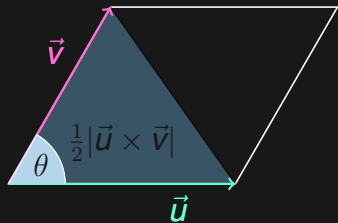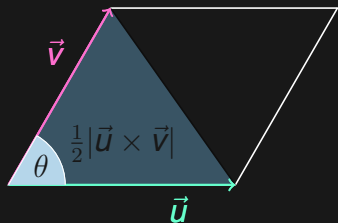
Cross product

▶ Allows us to calculate the area of the triangle formed by $\vec{u}$ and $\vec{v}$.

$$\frac{|\vec{u} \times \vec{v}|}{2}$$

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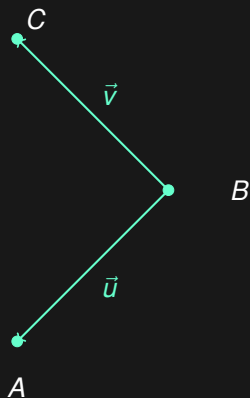$$\frac{|\vec{u} \times \vec{v}|}{2}$$

▸ And can tell us if the angle between $\vec{u}$ and $\vec{v}$ is positive or negative.

$$|\vec{u} \times \vec{v}| < 0 \quad \text{iff} \quad \theta < \pi$$
$$|\vec{u} \times \vec{v}| = 0 \quad \text{iff} \quad \theta = \pi$$
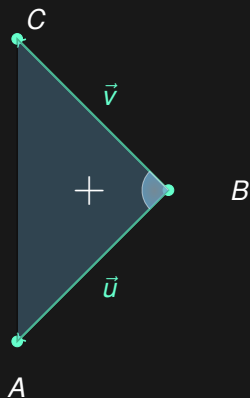$$|\vec{u} \times \vec{v}| > 0 \quad \text{iff} \quad \theta > \pi$$

Counterclockwise

- Given three points *A*, *B* and *C*, we want to know if they form a counter-clockwise angle in that order.

$$A \to B \to C$$

Counterclockwise

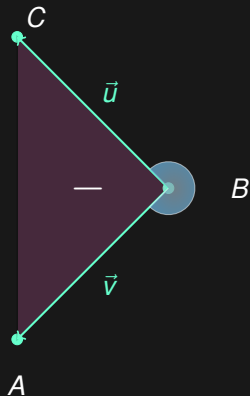- Given three points *A*, *B* and *C*, we want to know if they form a counter-clockwise angle in that order.

$$A \to B \to C$$

- We can examine the cross product of and the area of the triangle formed by

$$\vec{u} = B - C \quad \vec{v} = B - A$$
$$\vec{u} \times \vec{v} > 0$$

Counterclockwise

- The points in the reverse order do not form a counter clockwise angle.

$$C \to B \to A$$

- In the reverse order the vectors swap places

$$\vec{u} = B - A \quad \vec{v} = B - C$$
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▸ In the reverse order the vectors swap places

$$\vec{u} = B - A \quad \vec{v} = B - C$$
$$\vec{u} \times \vec{v} < 0$$

▸ If the points A, B and C are on the same line, then the area will be 0.

Counterclockwise

```
double cross(P(a), P(b)) {
    return real(a)*imag(b) - imag(a)*real(x);
}
double ccw(P(a), P(b), P(c)) {
    return cross(b - a, c - b);
}
bool collinear(P(a), P(b), P(c)) {
    return abs(ccw(a, b, c)) < EPS;
}
```

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Very common task is to find the intersection of two lines or line segments.

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- Given a pair of points (x_0, y_0), (x_1, y_1), representing a line we want to start by obtaining the form $Ax + By = C$.
- We can do so by setting

$$A = y_1 - y_0$$
$$B = x_0 - x_1$$
$$C = A \cdot x_0 + B \cdot y_1$$

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- We can do so by setting

$$A = y_1 - y_0$$
$$B = x_0 - x_1$$
$$C = A \cdot x_0 + B \cdot y_1$$

- If we have two lines given by such equations, we simply need to solve for the two unknowns, x and y.

Intersections

For two lines

$$A_0 x + B_0 y = C_0$$
$$A_1 x + B_1 y = C_1$$

The intersection point is

$$x = \frac{(B_1 \cdot C_0 - B_0 \cdot C_1)}{D}$$
$$y = \frac{(A_0 \cdot C_1 - A_1 \cdot C_0)}{D}$$

Where

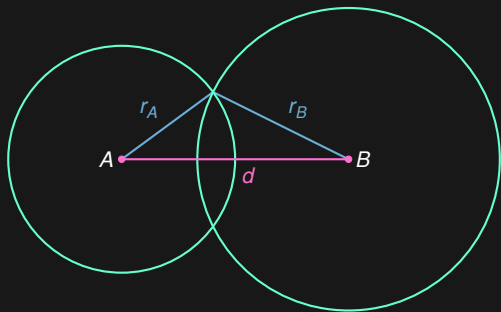$$D = A_0 \cdot B_1 - A_1 \cdot B_0$$

Intersections

Quite similar problem is to find the intersections of two circles.
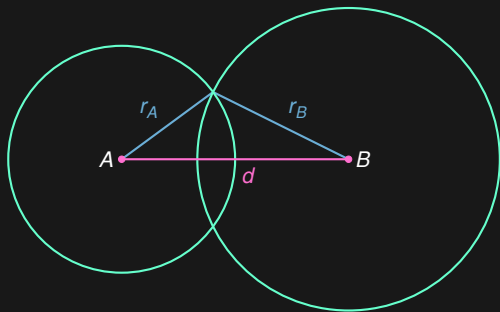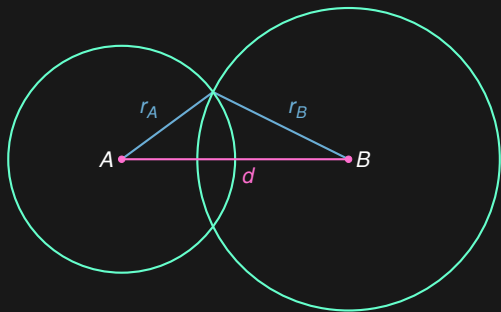
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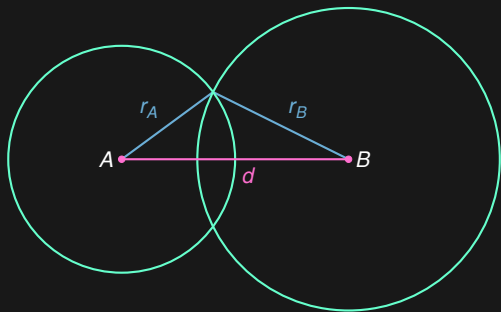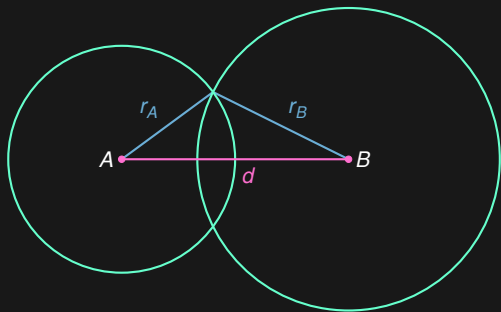- If $d > r_0 + r_1$ the circles do not intersect.

Intersections
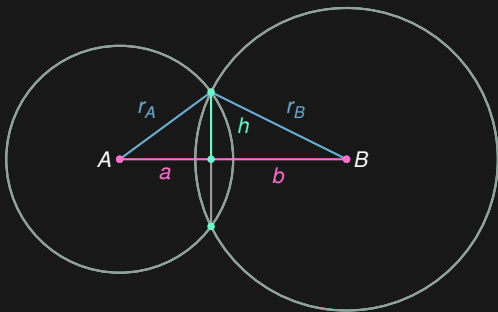
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- If $d < |r_0 - r_1|$, one circles is contained within the other.
- If $d = 0$ and $r_0 = r_1$, the circles are the same.
- Let's look at the last case.

Intersections

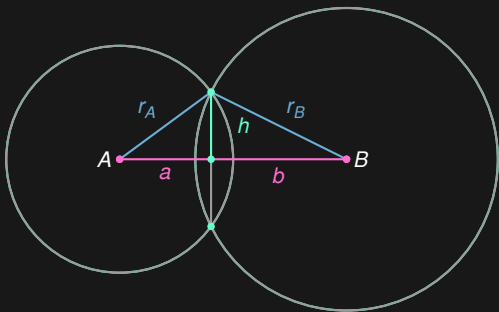Quite similar problem is to find the intersections of two circles.

▶ We can solve for the vectors a and h from the equations

$$a^2 + h^2 = r_0^2 \quad b^2 + h^2 = r_1^2$$

Intersections

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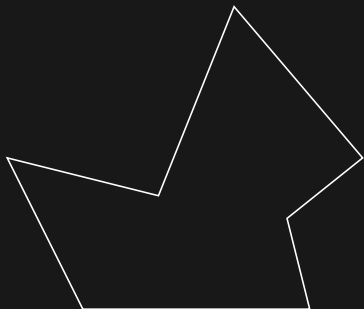$$a^2 + h^2 = r_0^2 \quad b^2 + h^2 = r_1^2$$

- We get

$$a = \frac{r_A^2 - r_B^2 + d^2)}{2 \cdot d}$$

$$h^2 = r_A^2 - a^2$$

Intersections

```
#define C(p, r) const point &p, double r
int intersect(C(A, rA), C(B, rB), point & res1, point & res2) {
    double d = abs(B - A);
    if ( rA + rB <  d - EPS || d < abs(rA - rB) - EPS){
      return 0;
    }
    double a = (rA*rA - rB*rB + d*d) / 2*d;
    double h = sqrt(rA*rA - a*a);
    point v = normalize(B - A, a);
    u = normalize(rotate(B-A), h);
    res1 = A + v + u;
    res2 = A + v - u;
    if (abs(u) < EPS){
        return 1;
    }
    return 2;
}
```
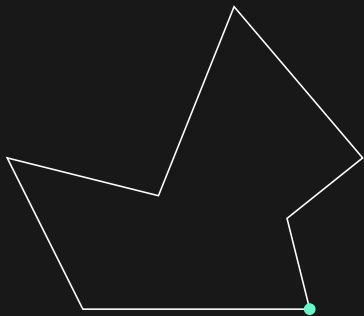
Polygons

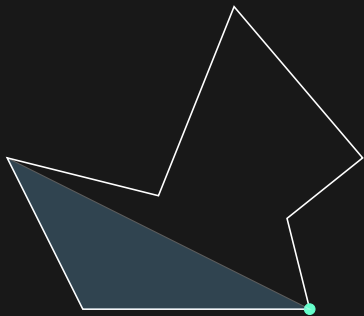▶ Polygons are represented by a list of points in the order representing the edges.

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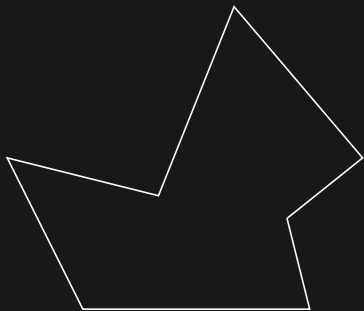
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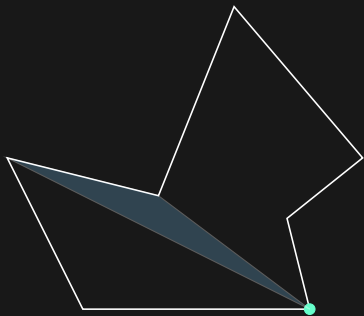- To calculate the area

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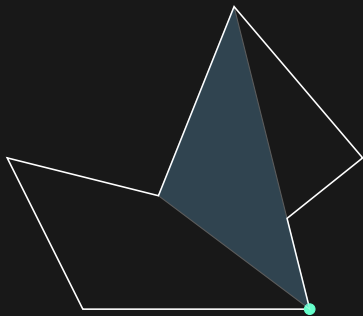- ▶ To calculate the area
  - We pick one starting point.
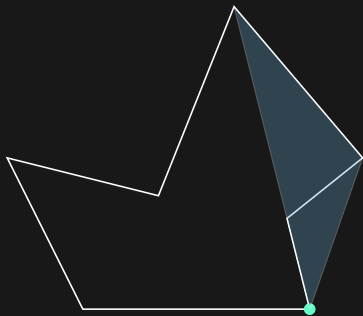
Polygons

- ► Polygons are represented by a list of points in the order representing the edges.
- ► To calculate the area
  - – We pick one starting point.
  - – Go through all the other adjacent pair of points and sum the area of the triangulation.

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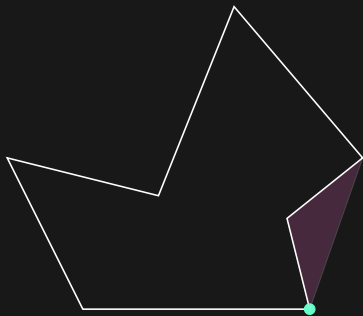
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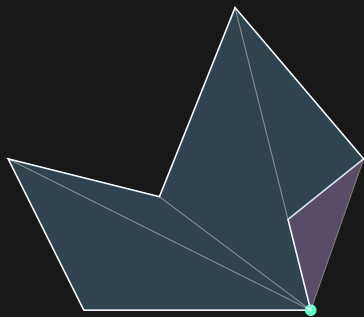
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  - Even if we sum up area outside the polygon, due to the cross product, it is subtracted later.

Polygons

```
double polygon_area_signed(const vector<point> &p) {
    double area = 0;
    int cnt = size(p);
    for (int i = 1; i + 1 < cnt; i++){
        area += cross(p[i] - p[0], p[i + 1] - p[0])/2;
    }
    return area;
}
double polygon_area(vector<point> &p) {
    return abs(polygon_area_signed(p));
}
```
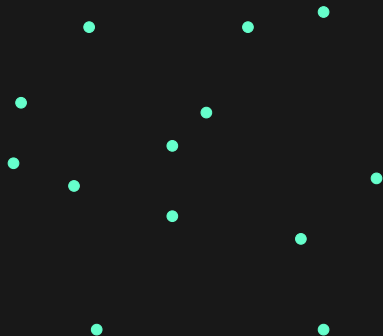
Convex hull

► Given a set of points, we want to find the convex hull of the points.

Convex hull

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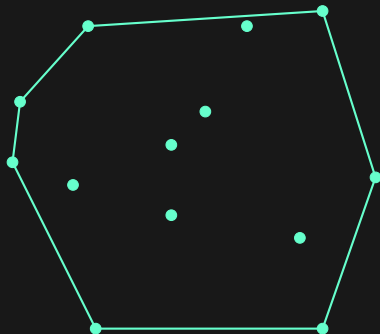
Convex hull

Graham scan:

Convex hull

Graham scan:

- Pick the point p_0 with the lowest y coordinate.

Convex hull

Graham scan:

- ► Pick the point p_0 with the lowest y coordinate.
- ► Sort all the points by polar angle with p_0.

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Graham scan:

- ▸ Pick the point p_0 with the lowest y coordinate.
- ▸ Sort all the points by polar angle with p_0.
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- ▸ If the current point forms a clockwise angle with the last two points, remove last point from the convex set.

Convex hull

Graham scan:

- Pick the point p_0 with the lowest *y* coordinate.
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- Otherwise, add the current point to the convex set.

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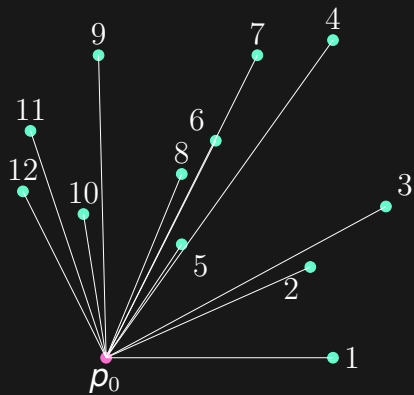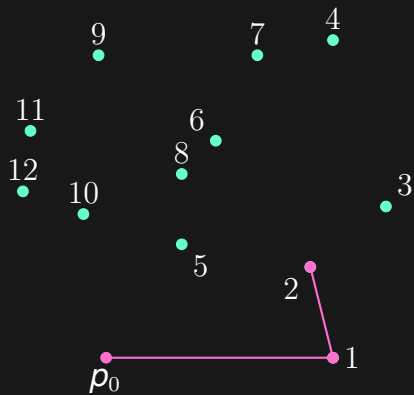Time complexity $O(N \log N)$.

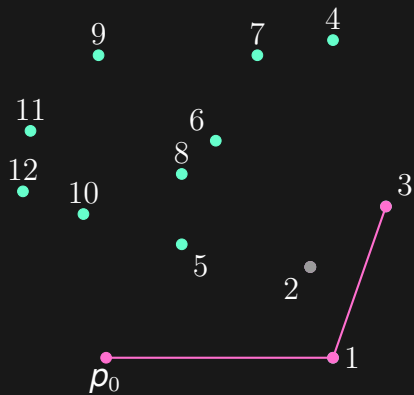# Convex hull
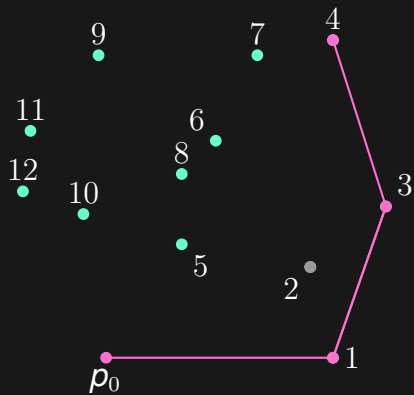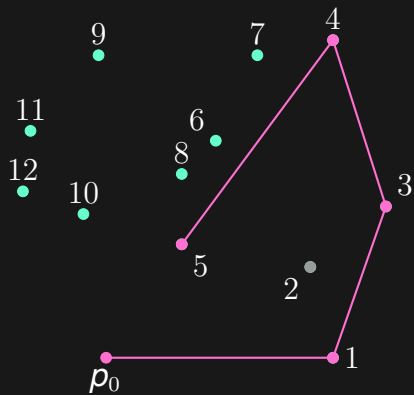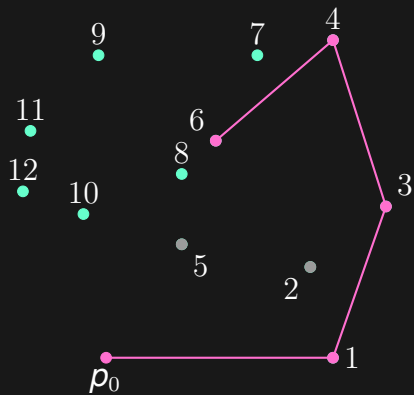
Convex hull
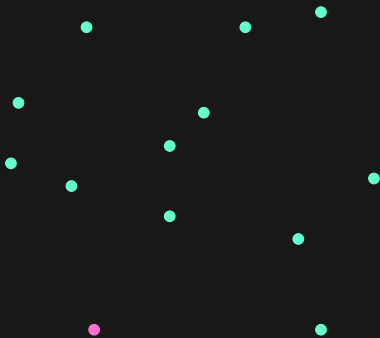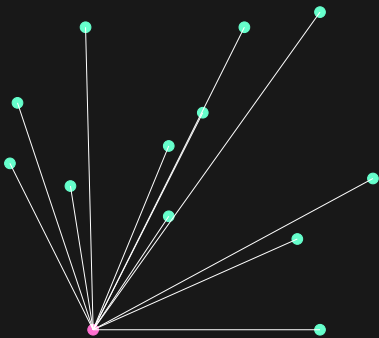
Convex hull

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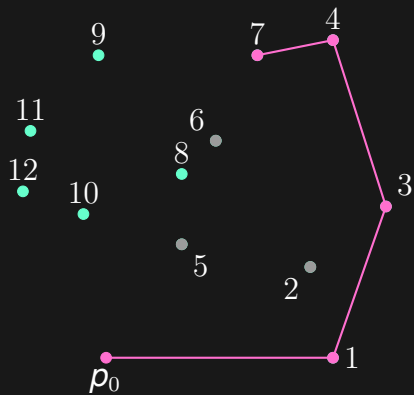# Convex hull

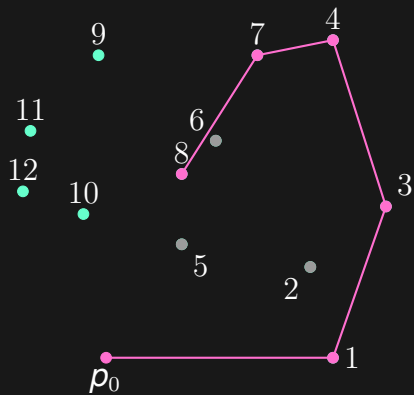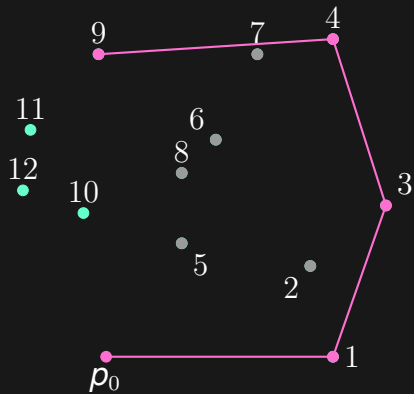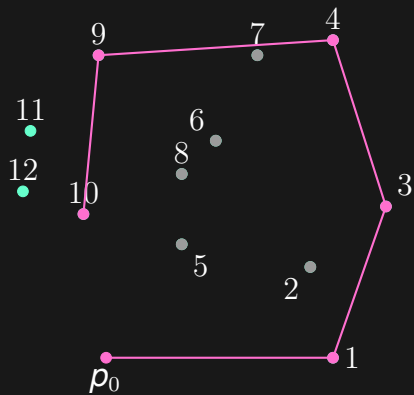# Convex hull

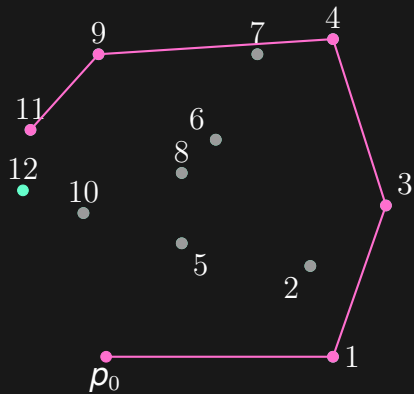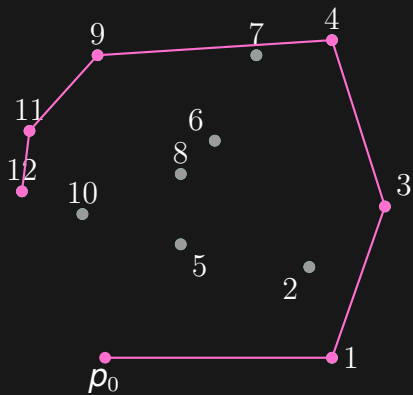# Convex hull

Convex hull

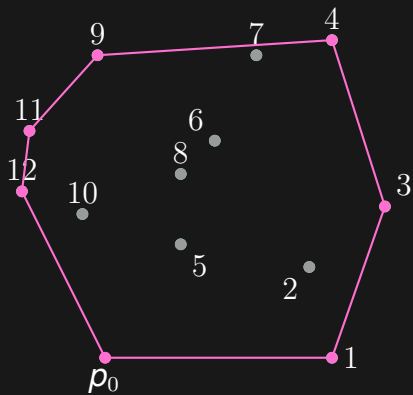# Convex hull

Convex hull

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Convex hull
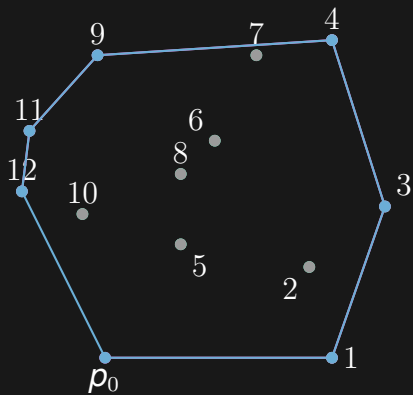
Convex hull

Convex hull

```
point hull[MAXN];
int convex_hull(vector<point> p) {
    int n = size(p), l = 0;
    sort(p.begin(), p.end(), cmp);
    for (int i = 0; i < n; i++) {
        if (i > 0 && p[i] == p[i - 1])
            continue;
        while (l >= 2 && ccw(hull[l - 2], hull[l - 1], p[i]) >= 0)
            l--;
        hull[l++] = p[i];
    }
    int r = l;
    for (int i = n - 2; i >= 0; i--) {
        if (p[i] == p[i + 1])
            continue;
        while (r - l >= 1 && ccw(hull[r - 2], hull[r - 1], p[i]) >= 0)
            r--;
        hull[r++] = p[i];
    }
    return l == 1 ? 1 : r - 1;
}
```

Convex hull

Many other algorithms exist

Convex hull

Many other algorithms exist
- Gift wrapping aka Jarvis march.

Convex hull

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- ▸ Quick hull, similar idea to quicksort.

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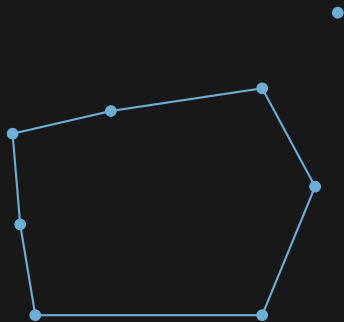
- ▶ Gift wrapping aka Jarvis march.
- ▶ Quick hull, similar idea to quicksort.
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Some can be extended to three dimensions, or higher.

Point in convex polygon

Simple algorithm to check if a point is in a convex polygon.

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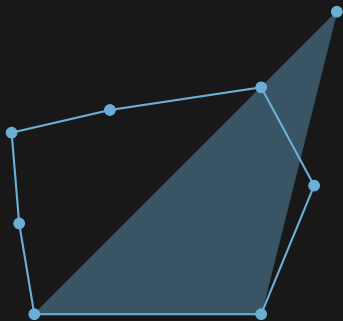- ▶ To check if our point is contained in the polygon we sum up the area of the triangles formed the point and every two adjacent points.

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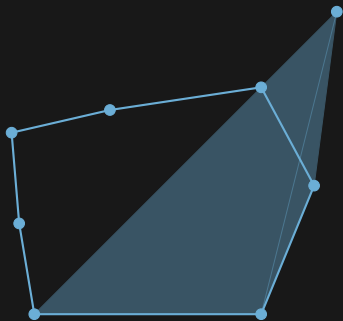
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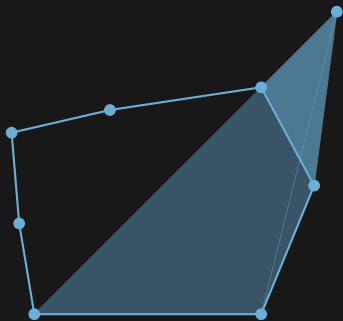
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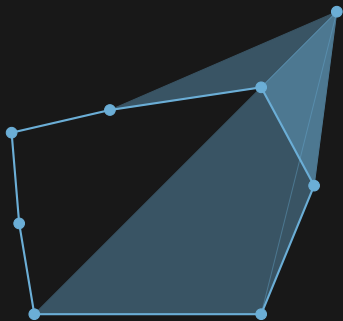
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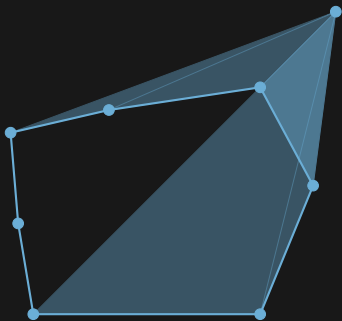
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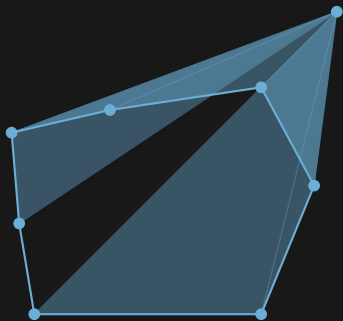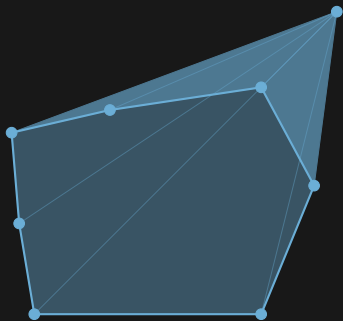
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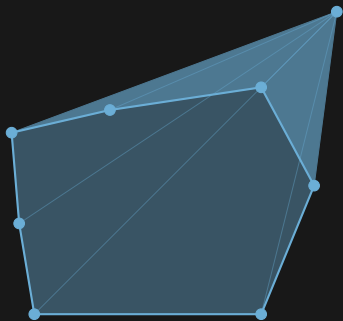
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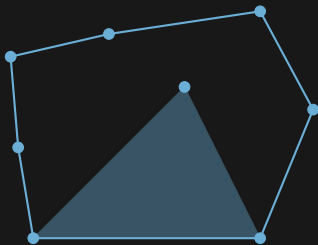
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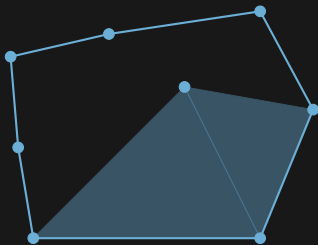
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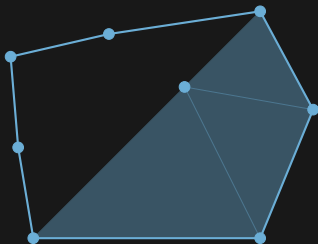
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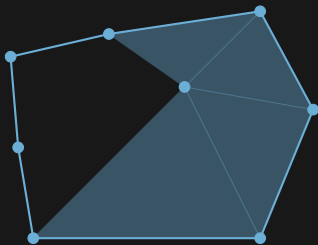
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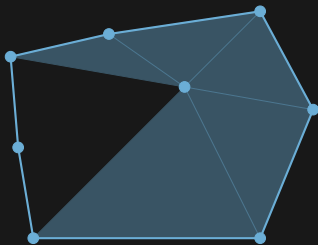
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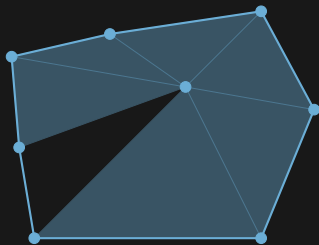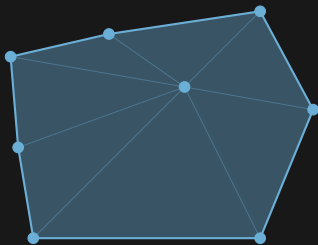
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Point in concave polygon

How about non convex polygon?

Point in concave polygon

How about non convex polygon?

- The *even-odd rule* algorithm.

Point in concave polygon

How about non convex polygon?

- The *even-odd rule* algorithm.
- We examine a ray passing through the polygon to the point.

Point in concave polygon

How about non convex polygon?

- The *even-odd rule* algorithm.
- We examine a ray passing through the polygon to the point.
- If the ray crosses the boundary of the polygon, then it alternately goes from outside to inside, and outside to inside.

Point in concave polygon
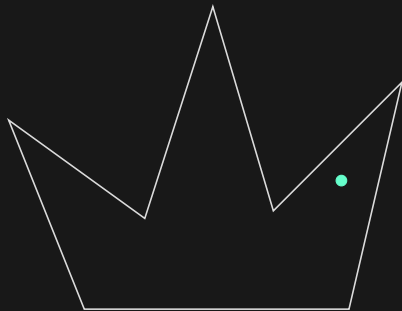
Point in concave polygon
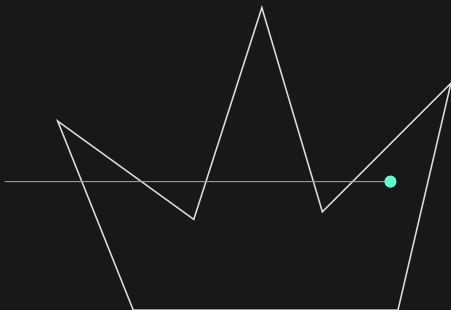
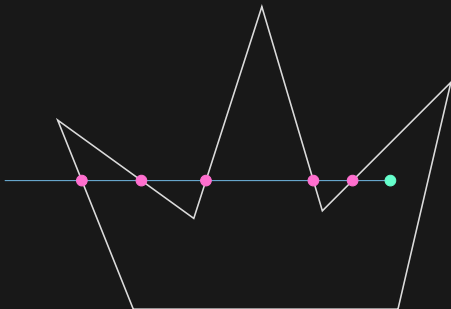▶ Ray from the outside of the polygon to the point.

Point in concave polygon
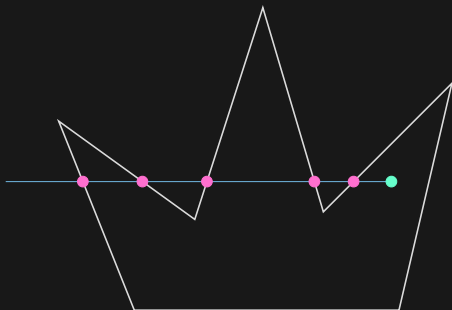
- ▶ Ray from the outside of the polygon to the point.
- ▶ Count the number of intersection points.

Point in concave polygon

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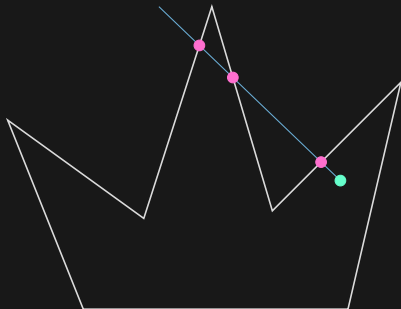
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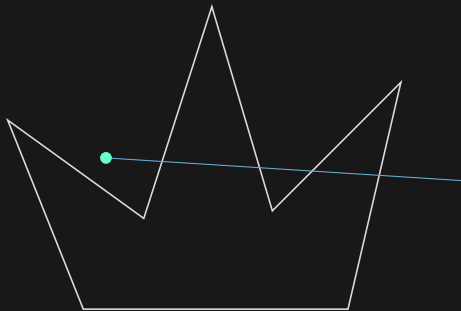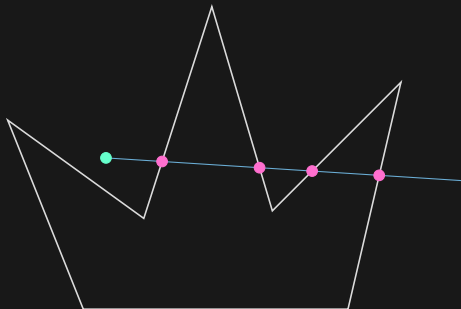
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Closest pair of points

Given a set of points, we want to find the pair of points with the smallest distance between them.

Divide and conquer algorithm;

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- Split the set into two equal sized sets by the vertical line of the medial x value.

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- Sort the two subsets by the *y*-coordinate.

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Divide and conquer algorithm;

- Sort points by the *x*-coordinate.
- Split the set into two equal sized sets by the vertical line of the medial *x* value.
- Solve the problem recursively in the left and right subset.
- Sort the two subsets by the *y*-coordinate.
- Find the smallest distance among the pair of points which lie on different sides of the line.

Closest pair of points

Takk fyrir önnina!

Gangi ykkur vel og góða skemmtun í prófinu á morgun!